G 5939 (Pages : 2) Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2017

First Semester

Faculty of Science

Branch I (A)—Mathematics

MT01 C04—GRAPH THEORY

(2012 Admission onwards)

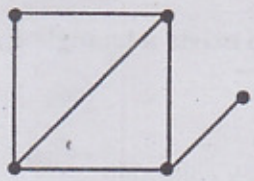
[Supplementary/Mercy Chance]

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries a weight of 1.

1. Show that in any group of n persons ($n \geq 2$), there are atleast two with the same number of friends.
2. Give an example of a non-simple disconnected graph with $\delta \geq \frac{n-1}{2}$.
3. Show that a tree with at least two vertices contains at least two pendant vertices.
4. Determine the number of spanning trees of the following graph :



5. Show that for any graph G, $\alpha + \beta = n$.
6. Prove that if G is k-critical then $\delta \geq k - 1$.
7. Show that the complement of a simple planar graph with 11 vertices is non-planar.
8. Draw the dual of the Herschel graph.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries a weight of 2.

9. Prove that every vertex of a disconnected tournament T with $n \geq 3$ is contained in a directed k-cycle, $3 \leq k \leq n$.

Turn over

10. Prove that the connectivity and edge connectivity of a simple cubic graph G are equal.
11. Prove that the number of edges in a tree with n vertices is $n-1$. Also prove that a connected graph with n vertices and $n-1$ edges is a tree.
12. Prove that if e is not a loop of G then $T(G) = T(G-e) + T(G \cdot e)$.
13. Prove that if for a simple 2-connected graph G, $\alpha \leq k$, then G is Hamiltonian.
14. (a) Define chromatic number of a graph.
    (b) Determine the chromatic number of (i) the Peterson graph ; (ii) the wheel W_n.
15. Show that a Hamiltonian cubic graph is 3-edge chromatic.
16. Prove that a graph G is planar if and only if each of its blocks is planar.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question carries a weight of 5.

17. (a) If the simple graphs G_1 and G_2 are isomorphic, prove that $L(G_1)$ and $L(G_2)$ are isomorphic.
    (b) Disprove H is a spanning subgraph of G implies that L(H) is a spanning subgraph of L(G).
18. State and prove Whitney's theorem on 2-connected graphs.
19. State and prove Cayley's formula for the number of spanning trees of a labeled complete graph.
20. Prove that a graph is Eulerian if and only if each edge e of G belongs to an odd number of cycles of G.
21. Prove that for every positive integer k, there exists a triangle-free graph with chromatic number k.
22. (a) Prove that $k_{3,3}$ is non-planar.
    (b) Prove that every planar graph is 6-vertex colorable.

$(3 \times 5 = 15)$